Effect of Grid Resolution on the Statistics of Passive Scalar in an Injection-Driven Channel

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Abstract. Effect of grid resolution on the statistics of passive scalar in a complex shear layer was investigated using a direct numerical simulation technique. The grid resolution in the shear layer which was generated from the interaction of main and injected streams strongly influences the subsequent evolution of the passive scalar. Dissipation, integral length-scale, skewness and flatness factors of the passive scalar are sensitive to the numerical resolution away from the wall where coherent structures grow very rapidly.

1 Introduction

Direct numerical simulation (DNS) resolves all the scales of fluid motion. Conceptually it is the simplest approach and, when it can be applied, it is most accurate and complete in the level of description provided. The drawback of DNS is of course its very large computational cost, and the fact that this cost increases rapidly with the Reynolds number. However, DNS studies have proved extremely valuable in many previous studies in revealing the physics of turbulence and turbulent flows.

For the turbulent flow in an injection-driven channel, an accurate prediction of the flow is of direct importance in many practical applications such as those in combustion chamber and transpiration cooling. Since turbulence plays a critical role in the evolution and dispersion of passive scalar, a better understanding of the flow characteristics will be very useful for the efficient design and operation of various thermal systems.

Even though a significant development has been made to the prediction method of velocity field in wall-bounded channel with transpired walls (Beddini [1]), relatively much less effort has been made in the calculation of passive scalar transport in spite

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of its practical importance. Many of earlier investigations had to rely on RANS type approach or relatively simple instrumentations and thus, an inherent limit was imposed in understanding the behavior of turbulent passive scalar field. As the flow situation becomes more complex, the Reynolds analogy between the flow and passive scalar deteriorates further and, thus, the analysis for the passive scalar becomes much more difficult. The present work mainly intended to examine the effect of numerical resolution on the passive scalar transport in complex flow situation. In line with the purpose of this study, a direct numerical simulation technique which gets solutions of governing equations without turbulence model was adopted.

The focus will be restricted to turbulence statistics such as turbulent Prandtl number, turbulent diffusivity and integral length-scale of the passive scalar which are usually hard to obtain directly from the measurements. Even though those statistics will require a significantly long averaging time, an attempt of getting perfectly smooth data by averaging over such a long period of time was not made in the present work. Instead, all the statistics were averaged over a time span which is sufficiently long enough only for up to the second-order statistics on the consideration of computational cost.

A brief description of numerical methodology will be provided in the next section and then, several statistical results will be discussed.

2 Description of Numerical Methodology

2.1 Governing Equations

Assuming that the flow is incompressible, the following non-dimensional equations for velocity and passive scalar were solved on a rectangular, staggered grid (Harlow & Welch [2]).

$$\frac{\partial u_i^*}{\partial x_i^*} = 0$$

$$\frac{\partial u_i^*}{\partial t^*} + \frac{\partial}{\partial x_j^*}(u_i^* u_j^*) = -\frac{\partial p^*}{\partial x_i^*} + \frac{1}{\mathrm{Re}_h}\frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*}$$

$$\frac{\partial T^*}{\partial t^*} + \frac{\partial}{\partial x_j^*}(u_j^* T^*) = \frac{1}{\mathrm{Re}_h \Pr}\frac{\partial^2 T^*}{\partial x_j^* \partial x_j^*}$$

All the variables are made dimensionless using an inlet bulk velocity and a half channel height. For convenience, the superscript * will be dropped hereinafter. The governing equations are integrated in time using a semi-implicit scheme. A low storage, 3rd order Runge-Kutta scheme was used for treating convective terms explicitly and the Crank-Nicolson scheme for viscous terms semi-implicitly. All the spatial derivatives are approximated with second order central difference scheme. For more numerical details, see Na [3].

2.2 Computational Details

Three-dimensional, rectangular computational domain consists of a streamwise extent of $26h$ and a spanwise extent of $6.5h$. Here h is the half channel height. The constant wall injection starts along both upper and lower wall from the location of x/h=13.4. The strength of wall injection, defined by the ratio of applied wall injection to the inlet bulk velocity, was set to 0.05. This is quite a strong injection and results in a strong acceleration of the flow in the main flow direction to satisfy global mass conservation. Thus, the pressure gradient (which does not appear in the mean concentration equation) is one of important terms in the mean momentum equation budget for the streamwise velocity component. The Reynolds number based on the inlet bulk velocity and a half channel height is 2250.

For the passive scalar, it was assumed that the bottom wall is maintained at a constant temperature (or concentration), $-T_w$, and the top wall at T_w. The Prandtl number was assumed to be 1 so that the working fluid can be thought of as a gas (instead of liquid). In order to prevent the numerical instability for the passive scalar, a widely used QUICK scheme (Leonard [4]) was employed for the convective terms of the passive scalar equation.

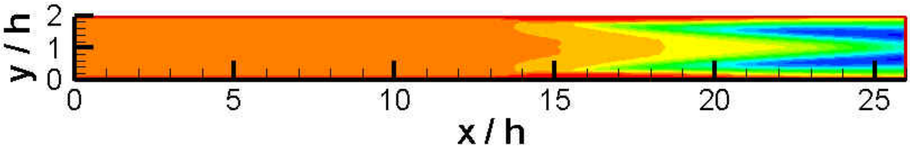
The no-slip boundary condition is used along the wall except in a region where constant blowing is applied. The flow is assumed to be homogeneous in the spanwise direction, justifying the use of periodic boundary condition in that direction.

In order to assess the effect of numerical resolution, a series of computations wereconducted and the test cases considered in the present paper are summarized in the following table.

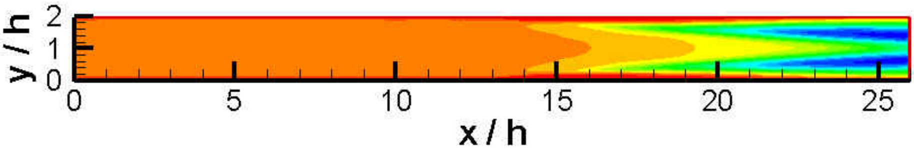
Table 1. Test cases performed

Name of test cases	Number of numerical grids
CASE1	257 x 129 x 129 ≈ 4.3 million grids
CASE2	257 x 257 x 129 ≈ 8.5 million grids
CASE3	513 x 257 x 129 ≈ 17 million grids
CASE4	513 x 257 x 257 ≈ 34 million grids

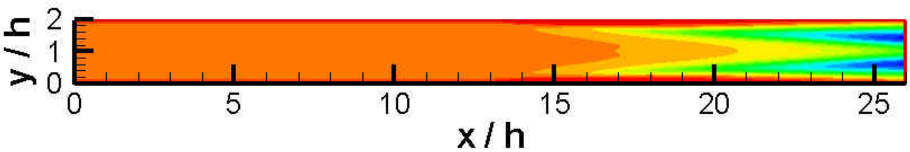
The geometry of the present injection-driven flow contains several regions of non-negligible gradients in the wall normal direction, which requires a very careful distribution of grid spacing. The CASE3 uses a 513x257x129 grid system and this gives the resolution of approximately $\Delta x^+ \approx 7.5$, $\Delta y^+_{\min} \approx 0.0056$, $\Delta y^+_{\max} \approx 1.8$, $\Delta z^+ \approx 7.5$ in terms of wall unit. When 257 grid points are used in the spanwise direction (CASE4), the grid spacing in the sapnwise direction becomes 3.8 in wall unit. Judging from the turbulence studies reported in the literature, it can be said that the present resolution is more than good for resolving the simple channel flow at comparable Reynolds number.



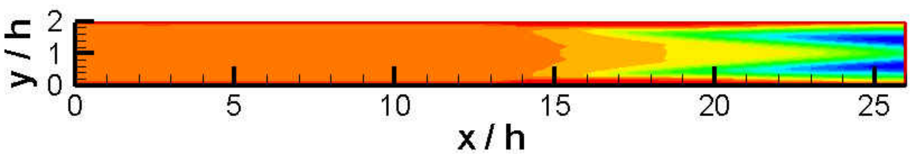
(a) CASE1



(b) CASE2



(c) CASE3



(d) CASE4

Fig. 1. Comparison of $-\overline{T'v'}$ contours obtained with different resolutions

3 Results and Discussion

The turbulent heat flux in the wall-normal direction is likely to be influenced most by the grid resolution in the shear layer formed in the middle of the channel. A series of contour plots in Figure 1 show the effect of resolution on $-\overline{T'v'}$ in (x-y) plane. Overall behavior is qualitatively similar but it definitely shows the differences especially in the shear layer after x/h>15. However, it can be said that, from an engineering point of view, the resolution of CASE1 is reasonably acceptable in catching the relevant flow physics. .

Turbulent diffusivity and turbulent Prandtl number at x/h=24 are compared in Figures 2-3. A non-negligible variation with the numerical resolution is realizable in both figures. Compared with the simple channel flow, turbulent diffusivities are significantly enhanced due to the turbulent activities in the developing shear layer. The turbulent Prandtl number also exhibits a qualitatively different distribution across the channel. Since new turbulent structures are being generated rapidly in the shear layer, sufficient resolution should be provided to capture the correct growth rate of those structures and thus, the grid resolution is thought to be responsible for the differences seen in these figures.

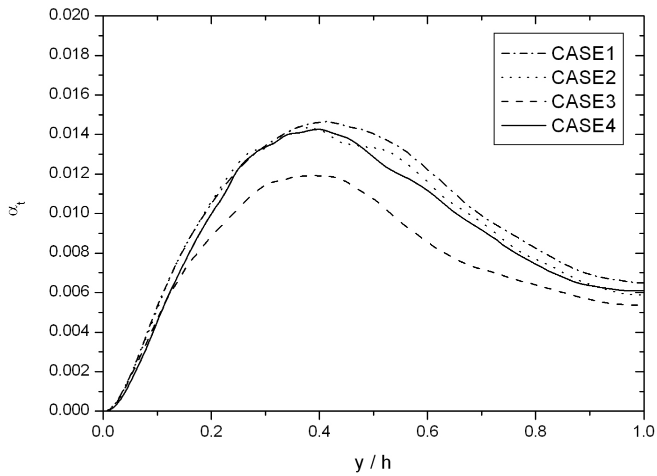


Fig. 2. Distribution of turbulent diffusivity at x/h=24

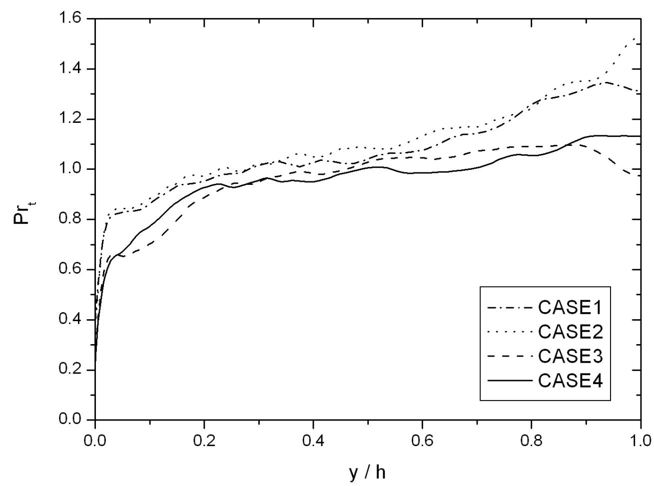


Fig. 3. Distribution of turbulent Prandtl number at x/h=24

A similar behavior of resolution-dependence can be seen in the distribution of dissipation of passive scalar. As the resolution gets better, smaller scale motions are better resolved and this results in a higher dissipation rate as shown in Figure 4. In

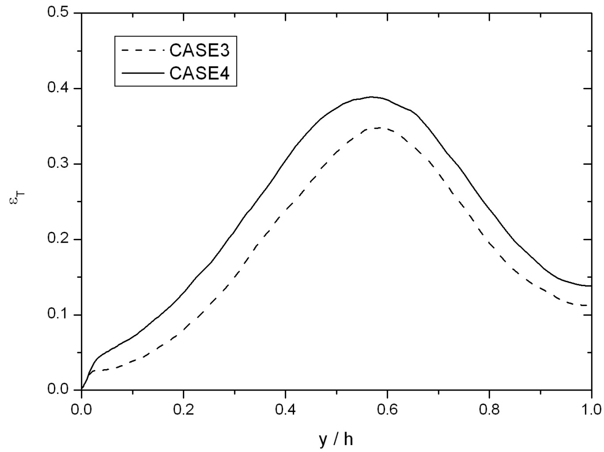


Fig. 4. Dissipation of passive scalar at x/h=24

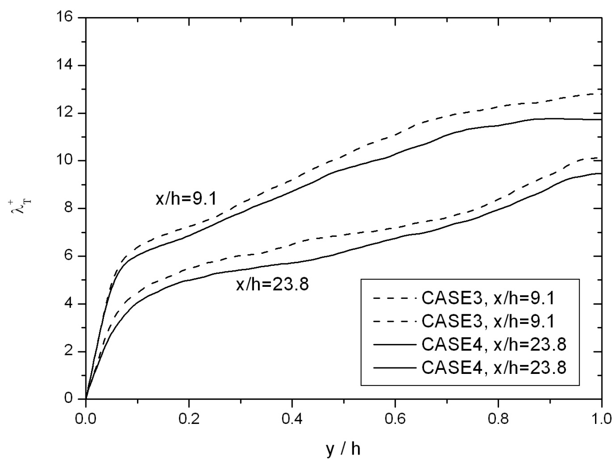
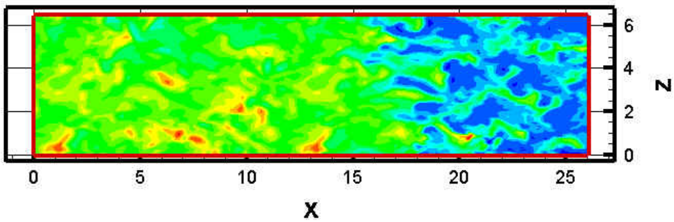
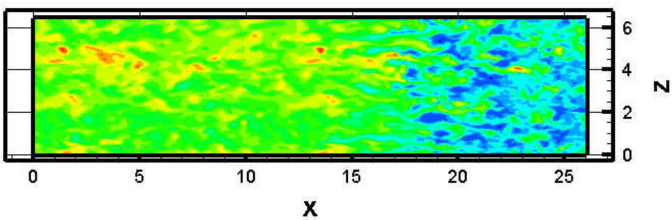


Fig. 5. Integral length-scale of passive scalar

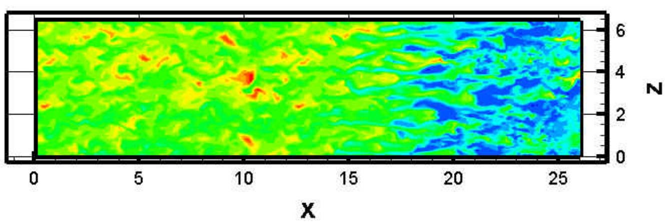
order to understand the turbulence maintenance mechanism, the correct prediction of dissipation is very important. Unfortunately, the simulation with more grid points than CASE4 was not possible due to the prohibitively high computational cost and it is not sure if the further improvement can be obtained with in Figure. Using the information of rms fluctuation and dissipation of passive scalar, one can define the integral length-scale shown in Figure 5. In the shear layer formed in the middle of the channel, the coherent structures which came from the upstream interact with the injected stream originating from the walls. In this process, the existing structures may be broken into smaller ones or smaller scale motions are newly born in this region. This possibility is illustrated in Figure 5. In any case, a better resolution gives a smaller length scales at different locations. This result can be also noticed in Figure 6 where instantaneous passive scalar fields are displayed in (x-z) plane at *y/h*=0.43.



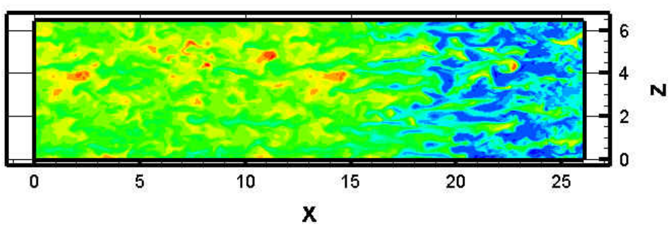
(a) CASE1



(b) CASE2



(c) CASE3



(d) CASE4

Fig. 6. Instantaneous concentration field in (x-z) plane at y/h=0.43

Finally, in Figures 7 and 8, skewness and flatness factors at *x/h*=24 obtained from CASE3 and CASE4 are compared. Note that they are calculated from 3rd and 4th order statistics, respectively. Both factors exhibit very high values close to the wall suggesting that the passive scalar is significantly disturbed (or modified by the wall injection). Skewness factor, which is a measure of intermittency, is significantly different from its Gaussian value of zero up to y/h=0.65. Flatness factor in the middle of the

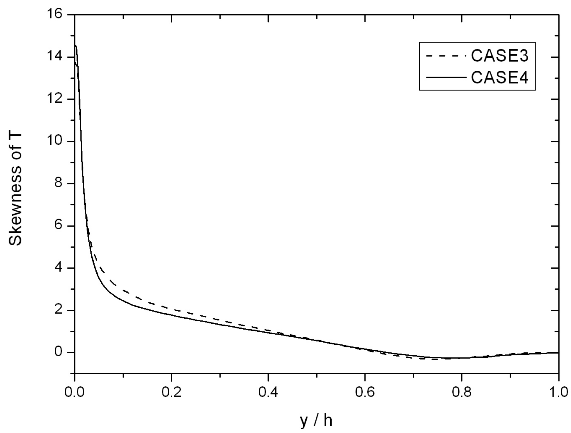


Fig. 7. Skewness factor of passive scalar

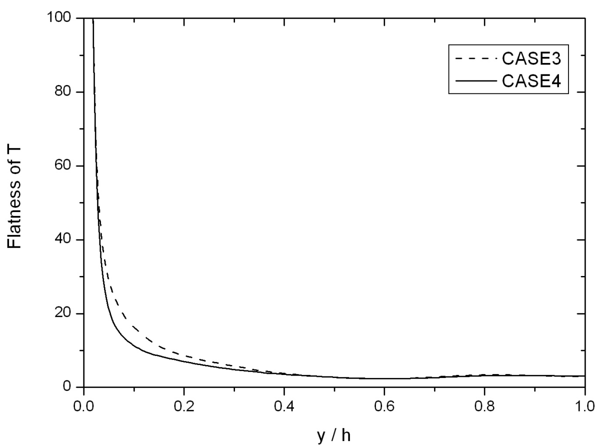


Fig. 8. Flatness factor of passive scalar

channel fluctuates around Gaussian value of 3 similar to the simple channel but it increases very rapidly in the vicinity of the wall. As expected from the previous results, Figures 7-8 show that the higher-order statistics between $0.05 < y/h < 0.4$ are also influenced by the resolution in the developing shear layer.

4 Summary

Direct numerical simulations were performed with different numerical resolutions in the injection-driven turbulent channel which is characterized by non-negligible streamwise inhomogeneity with an objective of analyzing the effect of grid resolution in complex flow situation. In the present configuration, the complexity of the flow came from the shear layer formed by the interaction of main flow with the wall injection. This inhomogeneity not only renders the computational analysis very expensive but also makes the quality of prediction be influenced by the grid resolution used.

A close investigation of the results suggests that the dissipation and integral length scale of passive scalar are strongly affected by the number of grids. When the resolution is sufficiently high, the turbulent Prandtl number turned out to be rather constant in the outer layer ($y/h > 0.2$). Regardless of the presence of large 2-D roller-type structures, which usually can be observed in the mixing layer, a proper resolution for the smaller scale motions present in the developing shear layer should be guaranteed for the accurate prediction of subsequent evolution of the passive scalar.

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References

1. Beddini, R. A.: Injection-Induced Flows in Porous-Walled Duct, AIAA J., vol. 24, no. 11 (1986) 1766-1772.
2. Harlow, F. H. and Welch, J. E.: Numerical Calculation of Time Dependent Viscous Incompressible Flow of Fluid with Free Surface, Phys. Fluids, vol. 8 (1965) 2182-2189.
3. Na, Y.: Direct Numerical Simulation of Turbulent Scalar Field in a Channel with Wall Injection. Numerical Heat Transfer, Part A (2005) 165-181.
4. Leonard, B. P.: A Stable and Accurate Convective Modeling Procedure Based on Quadratic Upstream Interpolation, Comput. Meth. Appl. Mech. Eng., vol. 19 (1979), 59-98.